AP Calculus
Practice and Review
Mr. James

Separable Differential Equations, Slope Fields and Euler Approximations

Separate the variables and solve the following differential equations.

1. $\frac{dy}{dx} = e^{x-y}$

2. $x^2(y^2+1)dx + y\sqrt{x^3+1}dy = 0$

3. $\sqrt{2xy}\frac{dy}{dx} = 1$

4. $\ln x \frac{dx}{dy} = \frac{x}{y}$

5. $(x+1)\frac{dy}{dx} = x(y^2+1)$

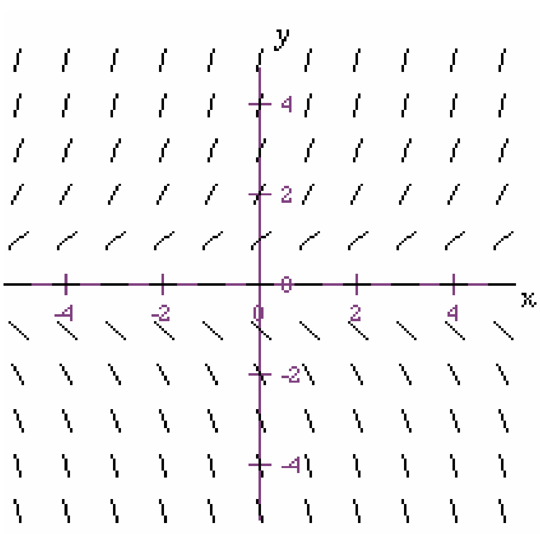
6. $x^2 y \frac{dy}{dx} = (x+1)\csc y$

If we interpret *dy*/*dx* as the slope of a tangent line, then at a point (*x*, *y*) on an integral curve of the equation *dy*/*dx* = *f*(*x*), the slope of the tangent line is *f*(*x*,*y*). What is interesting about this is that the slopes of the tangent lines to the integral curves can be obtained without actually solving the differential equation. For example, if

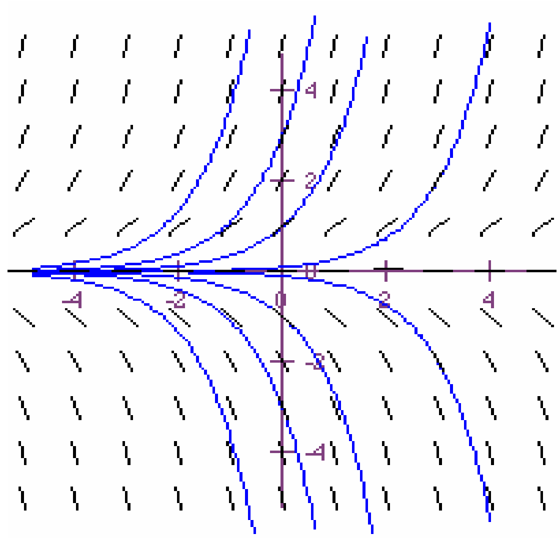
$$\frac{dy}{dx} = \sqrt{x^2+1}$$

then we know without solving the equation that at the point where $x = 1$ the tangent line to an integral curve has slope $\sqrt{1^2+1} = \sqrt{2}$; and more generally, at a point where $x = a$, the tangent to an integral curve has slope $\sqrt{a^2+1}$.

A geometric description of the integral curves of a differential equation *dy*/*dx* = *f*(*x*) can be obtained by choosing a rectangular grid of points in the *xy*-plane, calculating the slopes of the tangent lines to the integral curves at the lattice points, and drawing small portions of the tangent lines at those points. The resulting picture is called a ***slope field*** for the equation and shows the "direction" of the integral curves at the lattice points. Below is an example.



Slope Field for $\frac{dy}{dx} = y$



Some solutions for $\frac{dy}{dx} = y$

Referring to the above example, show that the following is true, and that the integral curves shown are solutions to the differential equation for different values of *C*.

$$\frac{dy}{dx} = y \Rightarrow Ce^{x}$$

Draw slope fields for the following differential equations. Next, sketch several solution curves over the slope field. Then solve the equation (remember that C is not determined) and compare your drawing to a graph of the solution.

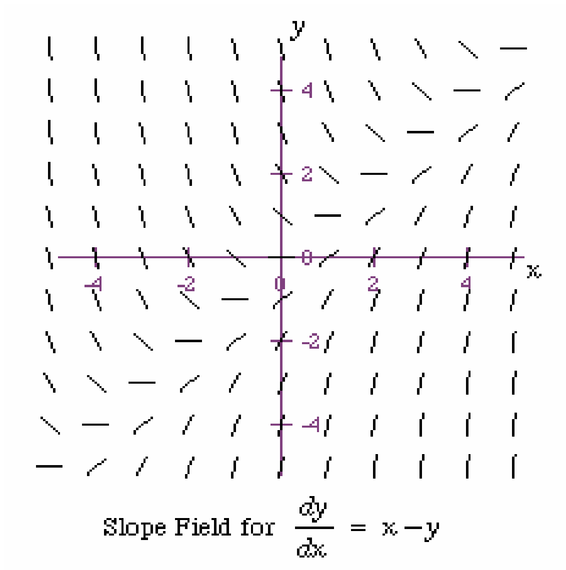
7. $\frac{dy}{dx} = 2x$

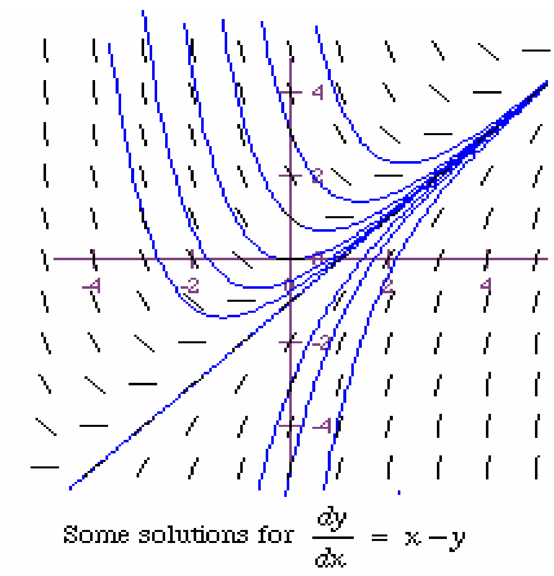
8. $\frac{dy}{dx} = x^2$

9. $\frac{dy}{dx} = -\frac{x}{y}$

10. $\frac{dy}{dx} = \frac{y}{x}$

Sometimes the differential equation may not be easily solvable, but the slope field will still allow us to sketch a family of solutions. Consider the following example. Then solve numbers 11 and 12 in a similar manner.





11. $\frac{dy}{dx} = x + y$ **12.** $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}$

13. For number 11, estimate the value of *y* when *x* is 2 on the solution curve containing (0, 1) using a step size $\Delta x = 0.5$.

14. For number 12, estimate the value of *y* when *x* is 2 on the solution curve containing (1, 1) using a step size $\Delta x = 0.25$.