BC Multiple Choice Sample Exam V  $*$ = on AB exam

* 1. $\frac{d}{dx}(2x^2+1)^4 = 4(2x^2+1)^3 \cdot 4x = 16x(2x^2+1)^3$  (E)

* 2. $\int x\sqrt{x^2+1}\,dx$  $\begin{matrix} u = x^2+1 \\ du = 2x\,dx \end{matrix} \rightarrow \frac{1}{2}\int u^{1/2}du = \frac{1}{3}u^{3/2} + C_1 = \frac{1}{3}(x^2+1)^{3/2} + C$  (C)

3. If $c$ is the only real number such that $f(c)=0$ then by the IVT, (A)

$f(a) < 0$ and $f(b) > 0$, $f(-2) = -3$ and $f(-1) = 3$.

4. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{2t+2}{2}\Big|_{t=1} = 2$. Also, $y(1) = 1^2 + 2(1) = 3$ and $x(1) = 2(1)+3 = 5$ (A)

$\therefore y - 3 = 2(x-5) \Rightarrow y = 2x - 7$

5. $\int_0^8 (8-x)^{-1/3}dx$  $\begin{matrix} u = 8-x & x=8 \rightarrow u=0 \\ du = -dx & x=0 \rightarrow u=8 \end{matrix} \rightarrow -\int_8^0 u^{-1/3}du = \int_0^8 u^{-1/3}du = \left[\frac{3u^{2/3}}{2}\right]_0^8 = 6$ (C)

6. $\int x\sin x\,dx \rightarrow$ $\begin{matrix} x & + & \sin x \\ 1 & - & -\cos x \\ 0 & & -\sin x \end{matrix} \rightarrow -x\cos x + \sin x + C$  (D)

* 7. I. $\frac{d}{dx}\int_0^3 f(x)dx = 0$, so I. is false ($2^{nd}$ FTC)  (B)

II $\int_3^x f'(x) = f(x) - f(3)$, so II is false (FTC)

III $\frac{d}{dx}\int_3^x f(x)dx = f(x)$ ($2^{nd}$ FTC), so III is true.

* 8. $\sin(xy) = x^2 \Rightarrow \frac{d}{dx}(\sin(xy) = x^2) \Rightarrow \cos(xy)(y + x\cdot y') = 2x$  (E)

$\Rightarrow y\cdot\cos(xy) + x\cdot\cos(xy)\cdot y' = 2x \Rightarrow y' = \frac{2x - y\cos(xy)}{x\cos(xy)} = \frac{2x\sec(xy) - y}{x}$

9. Concave up $\Rightarrow$ the rate of change is increasing: compare differences:  (B)

A

| $x$ | $g(x)$ | |
|---|---|---|
| 1 | -10 | > 3 |
| 2 | -7 | > 1 |
| 3 | -6 | > 4 |
| 4 | -2 | |

differences are not increasing

B

| $x$ | $g(x)$ | |
|---|---|---|
| 1 | 4 | > 2 |
| 2 | 6 | > 3 |
| 3 | 9 | > 5 |
| 4 | 14 | |

differences are increasing.

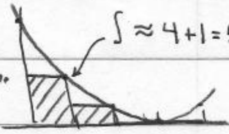
A similar analysis of C thru E shows that they are not concave up.

10. $\int \frac{dx}{x^2+4x} = \int \frac{A}{x}dx + \int \frac{B}{x+4}dx \Rightarrow A(x+4) + Bx = 1 \Rightarrow \begin{matrix} A = 1/4 \\ B = -1/4 \end{matrix}$  (E)

* 11. $x^2 - 6x + 9 = y$:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 9 | 4 | 1 | 0 | 1 |

"inscribed"




(D)

* 12. $y = \sin(2x) \rightarrow y' = 2\cos(2x) \rightarrow y'' = -4\sin(2x) \rightarrow y''' = -8\cos(2x)$  (B)

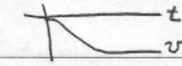
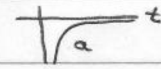
$\rightarrow y^{iv} = 16\sin(2x)$ $\therefore$ Even derivs: $y^{2n} = (-4)^n\sin(2x)$ $\therefore y^{20} = 2^{20}\sin(2x)$

[Incidentally, odd derivs are $y^{(2n+1)} = (-4)^n\cos(2x)$]

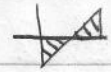
* 13. $f'(x) = 7 - 2x = 3 \Rightarrow x = 2$. $f(2) = 7(2) - (2)^2 = 10$  (B)

$\therefore y - 10 = 3(x-2) \Rightarrow y = 3x + 4$

* 14. The prompt only tells us that $f'(c)=0$ and $c$ is between (A)
a and b. I. $f(a)$ does not necessarily equal $f(b)$ - this is not Rolle's thm. II. c could be a POI and not an extremum (think $y=x^3$ @ $x=0$). III c need not be a POI.

* 15. The velocity graph looks something like  (D)
Acceleration is the derivative of $v$ and looks like 

16. $e^x \approx 1+x+\frac{x^2}{2}+\frac{x^3}{6} \Rightarrow e^{-4x} \approx 1+(-4x)+\frac{(-4x)^2}{2}+\frac{(-4x)^3}{6} = 1-4x+8x^2-\frac{32x^3}{3}$ (D)

17. $L_0^2 = \int_0^2 \sqrt{1+[f']^2}\,dx \quad f'=\sqrt{x^2-2x} \Rightarrow [f']^2 = x^2-2x \Rightarrow \sqrt{1+[f']^2} = \sqrt{x^2-2x+1} = |x-1|$ (D)
$\therefore L_0^2 = \int_0^2 |x-1|\,dx = 1$ :  ← The absolute value of the integral is 1

* 18. $\int_e^{e^2} \frac{dx}{x\ln x} \quad u=\ln x \quad du=\frac{1}{x}dx \quad x=e \to u=1 \quad x=e^2 \to u=2 \Rightarrow \int_1^2 \frac{1}{u}du = \left[\ln|u|\right]_1^2 = \ln 2 - \ln 1 = \ln 2$ (A)

* 19. $g(x)=f(3x) \Rightarrow g'(x)=f'(3x)\cdot 3 \Rightarrow g'(0.1)=3f'(0.3)=3(1.096)=3.288$ (E)

20. Both graphs are circles  The area is symmetric over $y=x$ (B)
of radius 1
$\therefore A = \int_{\theta_1}^{\theta_2} \frac{r^2}{2}d\theta = 2\int_0^{\pi/4} \frac{(2\sin\theta)^2}{2}d\theta = 4\int_0^{\pi/4} \sin^2\theta\,d\theta$

* 21. The prompt is the definition of the derivative applied to (D)
$f(x)=2x^5-5x^3 \Rightarrow f'(x)=10x^4-15x^2$

* 22. $\int_8^4 f(x)dx = -\int_4^8 f(x)dx = -\left[\int_2^8 f(x)dx - \int_2^4 f(x)dx\right] = -[-10-6] = 16$ (E)

* 23. $y'=3x^2+2ax+b \Rightarrow y''=6x+2a=0$ at $x=2 \Rightarrow 12+2a=0 \Rightarrow a=-6$. (D)
Then $y=x^3-6x^2+bx-8$ @ $(2,0) \Rightarrow (2)^3-6(2)^2+2b-8 \Rightarrow 24=2b \Rightarrow b=12$

24. $\langle x,y\rangle = \langle 4t^2, \sqrt{t}\rangle \Rightarrow \vec{v} = \langle 8t, \frac{1}{2}t^{-1/2}\rangle \Rightarrow \vec{a} = \langle 8, -\frac{1}{4}t^{-3/2}\rangle$. (B)
At $t=4$, $\vec{a} = \langle 8, -\frac{1}{4}(4)^{-3/2}\rangle = \langle 8, -\frac{1}{32}\rangle$

* 25. I & II are true (except perhaps in the debatable case of $f(x)=$ constant) (C)
III is not necessarily true (think absolute value type functions).

26. Ratio test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+2}\frac{x^{n+1}}{n+1}}{(-1)^{n+1}\frac{x^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{nx}{n+1} \right| < 1 \Rightarrow |x|<1$ (C)

$\Rightarrow -1<x<1$. Now, the end points: $x=-1$: $\Sigma = -1-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}$ (harmonic)
$x=1$: $\Sigma = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$ (alt, decreasing) converges! diverges ↑

27. $\sum_{n=0}^{\infty} \frac{(-1)^n(\pi)^{2n}}{(2n)!}$ is of the form $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ evaluated at $x=\pi$ (B)

This is the expansion of $\cos(x)$ $\therefore \sum (-1)^n(\pi)^{2n}/(2n)! = \cos(\pi) = -1$

* 28. $\frac{dy}{dx} = \frac{x}{y} \Rightarrow \int y\,dy = \int x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$ or $y^2 - x^2 = c$. At (A)

$(3,4)$, we have $4^2 - 3^2 = 7 = C$ $\therefore y^2 - x^2 = 7$ or $x^2 - y^2 = -7$

Calculator Active

* 29. I. $A = \int_0^a f(x)dx$ is true. II $A = \int_0^b f^{-1}(x)dx$ is true. (E)

III. $A = \int_0^b f^{-1}(y)dy$ is the same thing as II.

30. $\sum_{n=0}^{\infty} \frac{x^n}{2n} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}$; for $n=4 \Rightarrow \frac{1}{2\cdot 4} = \frac{f^{(4)}(0)}{4!} \Rightarrow f^{(4)} = 3$ (C)

* 31. $f'$ is a cubic: $f' = +1x^3 +$ stuff $\Rightarrow f = \int f'dx = \frac{1}{4}x^4 +$ stuff. (B)

So, $f$ is a quartic with positive leading coefficient.

TI-83 32. FTC: $f(5) = f(3) + \int_3^5 \frac{\sin(1+x^2)}{x^3-2x}dx = 7 + (-.0077) \approx 6.992$ (D)

33. $\sum_{n=1}^{\infty} |a_n|$ converges. I is true because either $a_n = |a_n|$, $a_n = -|a_n|$ (E)

or $a_n$ alternates; II is true by definition; III is true because

if $\sum a_n$ converges, $K\sum a_n$ converges.

*TI-83 34. 

$dV = (\text{area})(\text{thk}) = y^2 \cdot dx = 9(x-2)^4 dx$ (E)

$\therefore V = 9\int_0^2 (x-2)^4 dx = 57.6$

* TI-83 35. $y = \sin^2 x \rightarrow \frac{dy}{dx} = 2\sin x\cos x \Big|_{x=\pi/4} = 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 1 = \frac{dx}{dx} \Rightarrow k=1$ (C)

TI-83 36. $\mathcal{E}(x)_n < \left|\frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}\right| = \left|\frac{10000\cos(c)(1-0)^7}{7!}\right|$ The maximum value (E)

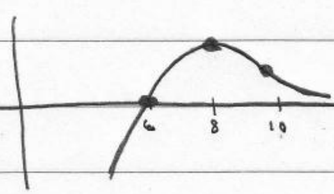
of $\cos(c)$ is 1, $\therefore \mathcal{E}(1)_6 < \left|\frac{10000}{7!}\right| = 1.984$

TI-83 37. FTC: $f(1) - f(0) = \int_0^1 \frac{\tan^2 x}{x^2+1}dx \Rightarrow f(0) = f(1) - \int_0^1 \frac{\tan^2 x}{1+x^2}dx = .5 - .345 = .155$ (B)

38. I $\Rightarrow$ zero at $x=6$

II $\Rightarrow$ max at $x=8$

III $\Rightarrow$ POI at $x=10$




A) $f'(0) < 0$ (D)

B) $f'(0) < f'(6) < 0$

C) $f''(4) \sim \frac{f'(6)-f'(0)}{6}$

D) $f''(10) = 0$ smallest

E) $f''(12) > 0$

Note: (C) is the smallest non-zero value.

*TI-83 39. New cost $= 14{,}500 + \int_0^5 (120 + 180\sqrt{t})\,dt = 16{,}441.64$ (C)

TI-83 40. $A = \int_0^k (2x - \sin x)\,dx = \left[x^2 + \cos x\right]_0^k = k^2 + \cos k - 1 = 0.1$ (A)

Graph $y = x^2 + \cos x - 1.1$ and find root between $0$ & $\frac{\pi}{2}$ : $k = 0.4436$

41. The values for the derivative increase until $x$ is near $1.000$, (C)

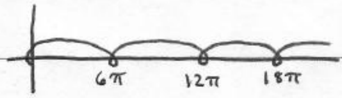
then the values of $f'$ decrease $\Rightarrow$ a P.O.I. exists near $1.000$

*TI-83 42. The "fifth week of growth" is from $t=4$ to $t=5$ : $\dfrac{m(5)-m(4)}{5-4} = 6.546$ g/wk (D)

43. Only (A) and (B) are positive, so $\int_0^2 f(x)dx > \int_0^1 f(x)dx$ (B)

TI-83 44. $\vec{v} = \langle 3 - 4\cos t, 4\sin t \rangle \Rightarrow \vec{s} = \langle 3t - 4\sin t, -4\cos t + 3 \rangle$ (E)

in order to start at $(0,-1)$. Graph for $t \in [0, 4\pi]$.




45. $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \cos(x_i)\,\Delta x = \int_0^\pi x\cos x\,dx = -2$ (A)