BC Multiple Choice Sample Exam IV $*$ = on AB exam

1. FTC: $y(x) - y(2) = \int_2^x y'(t)\,dt \Rightarrow y(x) = 4 + \int_2^x f(t)\,dt$ Ⓑ

2. $y' = 24x^5 - 24x^2 = 24x^2(x^3-1) = 0$

$$\begin{array}{l|ccccc} 24x^2 & + & 0 & + & + & + \\ x^3-1 & - & - & - & 0 & + \\ \hline y' & - & 0 & - & 0 & + \end{array}$$

min @ $x=1$ Ⓒ

3. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\sin t}{-4\cos t} \rightarrow$ undef @ $t = \frac{\pi}{2}, \frac{3\pi}{2}$. Then at $t = \frac{\pi}{2}$, $(x,y) =$ Ⓒ
$(3 - 4\sin\frac{\pi}{2}, 4 + \cos\frac{\pi}{2}) = (-1, 4)$; at $t = \frac{3\pi}{2}$, $(x,y) = (3 - 4\sin\frac{3\pi}{2}, 4 + \cos\frac{3\pi}{2}) = (7, 4)$

4. $x = 2t^2$, $y = t^3 \Rightarrow t = \sqrt{\frac{x}{2}} \Rightarrow y = \left(\frac{x}{2}\right)^{3/2} \therefore \frac{dy}{dx} = \frac{3}{2}\left(\frac{x}{2}\right)^{1/2} \cdot \frac{1}{2} = \frac{3}{4}\left(\frac{x}{2}\right)^{1/2}$ Ⓐ
and $\frac{d^2y}{dx^2} = \frac{3}{4} \cdot \frac{1}{2}\left(\frac{x}{2}\right)^{-1/2} \cdot \frac{1}{2} = \frac{3}{16}\left(\frac{x}{2}\right)^{-1/2}$. At $t = 3$, $x = 18 \therefore \frac{d^2y}{dx^2} = \frac{3}{16}\left(\frac{18}{2}\right)^{-1/2} = \frac{1}{16}$

5. Ratio Test: $\lim_{n\to\infty} \left| \dfrac{\frac{2^{n+1}x^{n+1}}{(n+1)}}{\frac{2^n x^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{2^{n+1}x^{n+1} \cdot n}{2^n x^n (n+1)} \right| = \lim_{n\to\infty} \left| \frac{2x\,n}{n+1} \right| = |2x| < 1$ Ⓔ

$\Rightarrow -1 < 2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$. End points: $x = -\frac{1}{2} \rightarrow \Sigma \Rightarrow 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ conv.
$x = +\frac{1}{2} \rightarrow \Sigma \Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges. $\therefore -\frac{1}{2} \le x < \frac{1}{2}$

$*$ 6. $x(t) = \ln t$ avg vel. is the average rate of change on $[1, e] = \frac{\ln e - \ln(1)}{e-1}$ Ⓒ
$= \frac{1}{e-1}$

7. Template for polar area: $A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta$ Ⓔ

8. Use tabular method: $\int 2x\cos(2x)dx = x\sin(2x) + \frac{1}{2}\cos(2x) + C$ Ⓓ

$*$ 9. $\int_{-3}^{3} |x+2|\,dx = \int_{-3}^{-2}(-x-2)dx + \int_{-2}^{3}(x+2)dx = -\left(\frac{x^2}{2} + 2x\right)\Big]_{-3}^{-2} + \left(\frac{x^2}{2} + 2x\right)\Big]_{-2}^{3} = 13$ Ⓒ

$*$ 10. $y = x^2 - 2kx$ has x-int at $x=0$ and $x = 2k$. Area $= \left| \int_0^{2k}(x^2 - 2kx)dx \right|$ Ⓑ
$= \left| \frac{x^3}{3} - kx^2 \right| \Big]_0^{2k} = \left| \frac{8k^3}{3} - 4k^3 \right| = \frac{4k^3}{3} = 36 \Rightarrow k^3 = 27 \therefore k = 3$

11. $\lim_{h\to 0} \frac{1}{h}\int_0^h \frac{\sin^2 t}{t^2}dt = \lim_{h\to 0} \frac{F(h) - F(0)}{h - 0} = F'(0)$ where $F'(t) = \frac{\sin^2 t}{t^2}$ Ⓒ
Then $F'(0) = \lim_{t\to 0} \frac{\sin^2 t}{t^2} = \lim_{t\to 0}\left(\frac{\sin t}{t}\right)^2 = \left(\lim_{t\to 0}\frac{\sin t}{t}\right) = (1)^2 = 1$

$*$ 12. MVT: $f'(c) = \frac{f(7) - f(-3)}{7 - (-3)} = \frac{2-4}{10} = \frac{-1}{5}$ Ⓑ

$*$ 13. $y = \frac{1-x}{x-1} = -1 \quad \therefore \frac{dy}{dx} = 0$ except at $x = 1$ Ⓑ

$*$ 14. $g(x) = \frac{1}{f(x)} = [f(x)]^{-1} \Rightarrow g'(x) = \frac{-1}{[f(x)]^2} \cdot f'(x) \Rightarrow g'(2) = \frac{-1}{[f(2)]^2} \cdot f'(2) = \frac{-1}{64} \cdot (-4) = \frac{1}{16}$ Ⓒ

15. I. $\sum_{n=1}^{\infty}\left(1 - \frac{4}{3}\right)^n = \Sigma\left(-\frac{1}{3}\right)^n$ is geometric w/ $|r| < 1 \rightarrow$ converges Ⓔ+
II. $\sum_{n=1}^{\infty}\left(1 + \frac{4}{3}\right)^n = \Sigma\left(\frac{7}{3}\right)$ is geometric w/ $|r| > 1 \rightarrow$ diverges
III. $\sum_{n=1}^{\infty}\left(1 + \frac{1}{n}\right)^n = e$ (converges)

16. Third degree term is $\frac{7(x-3)^3}{3!} = \frac{f'''(3)(x-3)^3}{3!} \Rightarrow f'''(3) = 7$ Ⓔ

$*$ 17. $f(x) = 15 - g(x) \Rightarrow f(x) - g(x) = 15 - 2g(x) \Rightarrow \int_{-2}^{2}[15 - 2g(x)]dx = 60 - 2\int_{-2}^{2} g(x)dx$ Ⓔ

$*$ 18. $\int_2^4 \left[\frac{d}{dt}(3t^2 + 2t - 1)\right]dt = (3t^2 + 2t - 1)\Big]_2^4 = 40$ Ⓑ

+ D&S Published answer key incorrectly lists A as the answer.

19. I. $\sum_{n=1}^{\infty} a_n$ converges because $a_n \le b_n \rightarrow$ true (D)

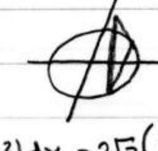
II $\sum_{n=1}^{\infty} c_n$ is unknown because $b_n \le c_n \rightarrow$ not necessarily true

III. $\sum_{n=1}^{\infty}(a_n+b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$ converges: sum of conv. series converge

20. $f(x) = \begin{cases} 1+e^{-x} & 0 \le x < 5 \\ 1+e^{x-10} & 5 \le x \le 10 \end{cases} \Rightarrow$ I. $\lim_{x\to 5^-} f(x) = 1+e^{-5}$, $\lim_{x\to 5^+} f(x) = 1+e^{5-10}$ $\Rightarrow$ continuous (true) (D)

II. $f'(x) = \begin{cases} -e^{-x} & 0 \le x < 5 \\ e^{x-10} & 5 \le x \le 10 \end{cases}$ $\lim_{x\to 5^-} f'(x) = -e^{-5}$, $\lim_{x\to 5^+} f'(x) = +e^{-5}$ $\Rightarrow$ not differentiable (false)

III. $f''(x) = \begin{cases} +e^{-x} & 0 \le x < 5 & >0 \\ e^{x-10} & 5 \le x \le 10 & >0 \end{cases}$ } concave up (true)

* 21. Base is $x^2+y^2=9 \Rightarrow y=\sqrt{9-x^2}$  $s = 2\sqrt{9-x^2}$ (E)

$dV = (\text{area})(\text{thk}) = \frac{\sqrt{3}}{4}s^2 \cdot (\text{thk})$

$dV = \sqrt{3}(9-x^2)dx \Rightarrow V = 2\sqrt{3}\int_0^3 (9-x^2)dx = 2\sqrt{3}\left(9x - \frac{x^3}{3}\right)\Big]_0^3 = 36\sqrt{3}$

* 22. $u = 25-x^2 \Rightarrow du = -2x\,dx$ $x=0 \rightarrow u=25$, $x=3 \rightarrow u=16$ (D)

$\therefore \int_0^3 x\sqrt{25-x^2}\,dx = \frac{1}{2}\int_{16}^{25}\sqrt{u}\,du$

23. $\lim_{n\to\infty}\left(\frac{(8-n)^{200}}{8^{n+2}}\right)\left(\frac{8^n}{(3-n)^{200}}\right) = \lim_{n\to\infty}\frac{1}{8^2}\frac{n^{200}}{n^{200}} = \frac{1}{64}$ (D)

* 24. Increasing at a decreasing rate means concave down. Choose... (C)

25. $\frac{dy}{dx} = y\left(8 - \frac{y}{1000}\right)$ has horiz. asymp when $\frac{dy}{dx} \rightarrow 0 \Rightarrow y=0, y=8000$ (E)

* 26. $\lim_{h\to 0}\frac{\tan(2(x+h)) - \tan(2x)}{h} = \frac{d}{dx}\tan(2x) = \sec^2(2x)\cdot 2$ (B)

27. I is false; the flow of the slope field is increasing. (D)

II is true; " " " " " " " "

III is true; the tangent lines near $x=0$ appear horizontal.

28. At A, f is decreasing and is concave up: $f'<0, f''>0 \therefore f'<f''$ (A)

At B, f is increasing and is concave down: $f'>0, f''<0 \therefore f'>f''$

At C, f appears to have a rel. max: $f'=0, f''<0 \therefore f'>f''$

Calculator active: TI-83

29. $e^{xy}=2 \Rightarrow \frac{d}{dx}e^{xy} = \frac{d}{dx}(2) \Rightarrow e^{xy}(y+x\cdot y')=0 \Rightarrow y' = \frac{-y}{x}\Big|_{x=1,\,y=\ln 2} = -\ln 2$ (A)

* TI-83 30. Speed $= |v(t)| = \left|\frac{dx}{dt}\right| = |4t^3-30t^2+58t-36|$, which has its max at $t=4$. (D)

* 31. f is always increasing, concave up for $x<0$, concave down for $x>0$. Choose... (E)

TI-83 32. $X(2) = X(1) + \int_1^2 2\sin t\,dt = 1+2.832 = 3.832$ (A)

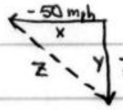
$y(2) = y(1) + \int_1^2 3\cos t\,dt = 1+2.728 = 3.728$ } $\Rightarrow (3.832, 3.728)$

* TI-83 33. $\int_1^{10}(2-\cos x)\,dx \approx \frac{1}{2}[2-\cos(1)+2-\cos(5)](5-1)+\frac{1}{2}[2-\cos(5)+2-\cos(8)](8-5)$ Ⓑ

$+\frac{1}{2}[2-\cos(8)+2-\cos(10)](10-8) = 17.129$

TI-83 34. $L = \int_0^1 \sqrt{1+(3x^2)^2}\,dx = 1.547$ Ⓓ

* TI-83 35. Graph f' on $[0,12]$. f' changes from + to − two times. Ⓒ

* TI-83 36. 

$z^2 = x^2+y^2 \Rightarrow x\frac{dx}{dt}+y\frac{dy}{dt} = z\frac{dz}{dt}$ Ⓓ

@ $t = 30$ min, $x=-25$, $y=-30$, $z=\sqrt{(-25)^2+(-30)^2} = 39.051$

Then $39.051\frac{dz}{dt} = -25(-50)-30(-60) \Rightarrow \frac{dz}{dt} = 78.102$ mph

* TI-83 37. Area under the parabola is $\int_0^2(-x^2+2x)\,dx = \frac{4}{3}$. Area of square is 4. Ⓒ

Area above the parabola is $4-\frac{4}{3}=\frac{8}{3}$ $\therefore$ Prob $= \frac{8/3}{4} = \frac{2}{3}$

38. $f'' = 5(x^2+5)^4\cdot 2x = 0$ at $x=0$ only. Ⓑ

39. $f(x) = \int_0^x f'(t)\,dt = C + \frac{3x^2}{2} - \frac{9x^4}{8} + \frac{81x^6}{240} - \frac{3x^8}{480} + \cdots$ $f(0) = 2$ Ⓔ

$\Rightarrow f(x) = 2 + \frac{3x^2}{2} - \frac{9x^2}{8} + \frac{27x^6}{80} - \frac{3x^8}{480} + \cdots$

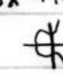
TI-83 40. Graph the "race" in parametric mode, simultaneous plot with Ⓐ

$T\in[0,2\pi]$, $X\in[0,18.9]$, $y\in[-10,10]$. Beta wins.

41. $\frac{dy}{dx} = e^x y \Rightarrow \frac{1}{y}dy = e^x dx \Rightarrow \ln|y| = e^x + C \Rightarrow |y| = Ae^{e^x}$ using $(0,3)$, Ⓔ

$3 = Ae^{e^0} \Rightarrow A = \frac{3}{e}$ $\therefore$ $y = \frac{3e^{e^x}}{e}$

TI-83 42. $y^2-x^3-15x^2=10 \Rightarrow y = \pm\sqrt{x^3+15x^2+10} \Rightarrow y' = \pm\frac{1}{2}(x^3+15x^2+10)^{-\frac{1}{2}}(3x^2+30x)$ Ⓓ

$= \pm\frac{3}{2}(x^3+15x^2+10)\cdot x(x+10) = 0$ at $x=0,-10$. Graph: 

TI-83 43. $\frac{dy}{dt} = ky \Rightarrow \frac{1}{y}dy = k\,dt \Rightarrow \ln y = kt+C \Rightarrow y = Ae^{kt}$. At $t=20$, Ⓓ

$y=\frac{1}{2}A \Rightarrow \frac{1}{2} = e^{20k} \Rightarrow \ln\frac{1}{2} = 20k \Rightarrow k = \frac{1}{20}\ln(\frac{1}{2}) = -0.034$

44. $P_{2,2}(x) = a + b(x-2) + c(x-2)$. The graph indicates that Ⓐ

$f(2)\approx -4 = a$; $f'(2) = 0 = b$; $f''(2)>0$ (concave up). $\therefore$ $a<0$, $b=0$, $c>0$

TI-83 45. Tabular: Ⓔ

x	y	Δx	$\frac{dy}{dx} = \frac{-x^2}{y}$	$\Delta y = \frac{dy}{dx}\Delta x$
3	−2	−.3	$\frac{-(3)^2}{-2} = \frac{9}{2}$	−1.35
2.7	−3.35			