BC Multiple Choice Sample Exam III $\qquad$ * = on AB exam

* 1. $A = \int_1^3 (3x^2+2x)\,dx = [x^3+x^2]_1^3 = (3^3+3^2)-(1^3+1^2) = 34$ (B)

2. Eliminate the parameter t: $y = 1-\cos 2t = 1-(2\cos^2 t-1) = 2-2\cos^2 t$ (B)

$\therefore$ since $x=\cos t$, $y = 2-2x^2$, a parabola.

3. $y = 2xe^{-x} \Rightarrow y' = 2e^{-x}(1-x) \Rightarrow y'' = 2e^{-x}(x-2) = 0$ when $x=2$ (D)

4. $\langle x,y\rangle = \langle e^t\sin t, e^t\cos t\rangle \Rightarrow \langle \frac{dx}{dt}, \frac{dy}{dt}\rangle = \langle e^t(\cos t-\sin t), e^t(\cos t+\sin t)\rangle$ (E)

evaluated at π is $\langle -e^{\pi}, -e^{\pi}\rangle$

* 5. $v(t) = t^2 \Rightarrow x(3)-x(1) = \int_1^3 t^2\,dt = \left[\frac{t^3}{3}\right]_1^3 = \frac{27}{3}-\frac{1}{3} = \frac{26}{3}$ (C)

6. $\frac{d}{dx}\left[(4x^2)\cos x\right] = 8x\cos x - 4x^2\sin x$ (B)

* 7. I is true; the contours for $y>2$ flow to the horiz. asymp $y=2$ (D)

II is true; " " " $y>0$ " " " " " "

III is false; " " " $y<0$ flow to $y\to-\infty$

* 8. $\int_1^3 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_2^{10}\frac{1}{u}\,du$ $\quad u=1+x^2$ $du = 2x\,dx$ $x=1\to u=2$; $x=3\to u=10$ (D)

$= \frac{1}{2}\ln|u|\Big]_2^{10} = \frac{1}{2}(\ln 10-\ln 2) = \frac{1}{2}\ln 5 = \ln\sqrt{5}$

* 9. $\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right) = \frac{d}{dx}\ln(x^2-1)^{-1} = -\frac{d}{dx}\ln(x^2-1) = \frac{-1}{x^2-1}\cdot 2x = \frac{-2x}{x^2-1}$ (B)

10. $\int_1^\infty x^{-5/4}\,dx = \lim_{k\to\infty} -4x^{-1/4}\Big]_1^\infty = -\lim_{k\to\infty}4k^{-1/4}+4(1)^{-1/4} = 4$ (A)

* 11. $u = x^2+1 \Rightarrow x^2 = u-1$ $\quad du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}du$ (A)

Then $\int_0^2 \frac{x^3}{x^2+1}\,dx = \frac{1}{2}\int_1^5 \frac{u-1}{2u}\,du$

* 12. $y' = \frac{k(k+x)-1(kx+8)}{(k+x)^2} = \frac{k^2+\cancel{kx}-\cancel{kx}-8}{(k+x)^2}\Big|_{x=-2} = 1$ (slope of $y=x+4$ (D)

$\Rightarrow k^2-4k+4 = k^2-8 \Rightarrow -4k = -12 \Rightarrow k=3$

13. $\int_0^k (4kx-5k)\,dx = [2kx^2-5kx]_0^k = 2k^3-5k^2 = k^2 \Rightarrow 2k^3-6k^2 = 0$ (C)

$\Rightarrow 2k^2(k-3) = 0 \Rightarrow k = \cancel{0}$ or 3 $\quad$ not positive

14. $\sum_{k=0}^{\infty}\left(-\frac{\pi}{3}\right)^k$ is geometric notice that $|r| = \frac{\pi}{3} > 1 \to$ diverges (E)

* 15. $y = 5^{x^3-2} \Rightarrow \frac{dy}{dx} = \ln 5\cdot 5^{x^3-2}\cdot 3x^2$ (B)

* 16. Here $\Delta x \leftrightarrow \frac{1}{n}$, so we are adding $\sin\pi\cdot\Delta x + \sin\pi\cdot 2\Delta x + \cdots \sin\pi$ (B)

which is the Riemann sum for $\int_0^1 \sin\pi x\,dx = \frac{1}{\pi}\left[-\cos\pi x\right]_0^1 = \frac{2}{\pi}$

* 17. f' shows that f decreases for $x<2$ and increases from $x=2$ (D)

to ∞ with a P.O.T at $x=1$. Choose...

18. $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \left(\frac{2}{2\sqrt{2t+5}}\Big/(1-2t)\right)\Big|_{t=2} = \left(\frac{1}{9}\Big/-1\right) = -\frac{1}{9}$ (A)

* 19. MVT: $\frac{f(10)-f(-2)}{(10)-(-2)} = \frac{2-26}{10+2} = -2$ $\quad f' = -2$ at $(4,23)$ $\therefore y-23 = -2(x-4)$ (C)

$\Rightarrow y = -2x+31$

* 20. $\frac{dy}{dx} = 4xy \Rightarrow \frac{1}{y}dy = 4x\,dx \Rightarrow \ln|y| = 2x^2 + C \Rightarrow |y| = Ae^{2x^2}$ Ⓔ

Using $(0,4)$, $|4| = Ae^0 \Rightarrow A = 4 \quad \therefore y = 4e^{2x^2}$

21. $\int \frac{dx}{x^2-9} = \int \left(\frac{A}{x+3} + \frac{B}{x-3}\right)dx \Rightarrow Ax - 3A + Bx + 3B = 1 \Rightarrow \begin{matrix} A = -1/6 \\ B = 1/6 \end{matrix}$ Ⓔ

$\therefore \quad = \frac{1}{6}\int -\frac{1}{x+3} + \frac{1}{x-3}\,dx = -\frac{1}{6}\ln|x+3| + \frac{1}{6}\ln|x-3| + C = \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + C$

22. $y' = e^{e^x} \cdot e^x = e^{(x+e^x)} \quad (y')^2 = e^{2(x+e^x)} \quad \therefore L = \int_0^1 \sqrt{1 + e^{2(x+e^x)}}$ Ⓐ

23. $\frac{dx}{dt} = x,\ \frac{d}{dt}\left(\frac{1}{x}\right) = -\frac{1}{x^2} \cdot \frac{dx}{dt} = -\frac{1}{x^2} \cdot x = -\frac{1}{x}\Big|_{x=\frac{1}{4}} = -4$ Ⓑ

* 24. $f(x) = (e^{2x}+1)^{1/2} \Rightarrow f'(x) = \frac{1}{2}(e^{2x}+1)^{-1/2} \cdot e^{2x} \cdot 2$ Ⓒ

$\therefore f(0) = \frac{1}{2}(e^0+1)^{-1/2} \cdot e^0 \cdot 2 = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

* 25. $x^2y + yx^2 = 2x^2y = 6 \Rightarrow x^2y = 3 \Rightarrow 2xy + x^2 \cdot y' = 0 \Rightarrow y' = \frac{-2y}{x}$ Ⓔ

Then $y'' = \frac{-2xy' + 2y}{x^2} = \frac{4y + 2y}{x^2} = \frac{6y}{x^2}$. Then, at $(1,3)$, $y'' = \frac{6(3)}{(1)^2} = 18$

* 26. 

$x + 2y = 4 \quad dV = (\text{area})(\text{thk}) = \frac{1}{2}\pi r^2(\text{thk}) = \frac{\pi}{2} \cdot \frac{1}{4}\left(2 - \frac{x}{2}\right)^2 dx$ Ⓐ

$\Rightarrow y = 2 - \frac{x}{2} \quad \therefore V = \frac{\pi}{8}\int_0^4 \left(4 - 2x + \frac{x^2}{4}\right)dx = \frac{\pi}{8}\left(4x - x^2 + \frac{x^3}{12}\right)\Big|_0^4 = \frac{2\pi}{3}$

27. I. $\sum_{n=1}^{\infty}(-1)^{n+1} = 1 - 1 + 1 - 1 + \cdots$ diverges Ⓐ

II. $\sum_{n=1}^{\infty}(-1)^{n+1} = 1 - 2 + 3 - 4 + \cdots$ diverges

III. $\sum\left(\frac{1+n}{n}\right)^n \Rightarrow \lim_{n\to\infty}\left(\frac{1+n}{n}\right)^n \to 1$ diverges

28. $\lim_{x\to a}\frac{x^4 - a^4}{x^2 - a^2} = \lim_{x\to a}\frac{(x^2+a^2)\cancel{(x^2-a^2)}}{\cancel{(x^2-a^2)}} = 2a^2 = 16 \Rightarrow a^2 = 8 \Rightarrow a = 2\sqrt{2}$ Ⓑ

Calculator Active: TI-83

29. $\frac{dr}{d\theta} = -5\sin\theta = 0$ @ $\theta = 0, \pi, 2\pi$ on $[0,8]$ r': $0 \; - \; \pi \; + \; 2\pi \; - \; 8$ (rel min. at π) Ⓑ

TI-83 30. Error bound. On $[0, \frac{\pi}{2}]$ the largest $f^{(n)}$ can be is $1 \Rightarrow M = 1$ Ⓒ

Then $\frac{1\left(\frac{\pi}{2}\right)^{2n}}{(2n)!} < \frac{1}{10000} \Rightarrow$ Guess $2n = 8 \to \frac{(\pi/2)^8}{8!} = .0009 > \frac{1}{10000}$.

Guess $2n = 10 \to \frac{(\pi/2)^{10}}{10!} = .000025 < \frac{1}{10{,}000}$ $\therefore$ We need 6 terms.

31. $f'(x) = 2x - 5\sin x \Rightarrow f''(x) = 2 - 5\cos x = 0$ when $\cos x = \frac{2}{5}$. Ⓔ

This occurs twice on the unit circle. So $0 \le x \le 2n$ is n times around the unit circle. $\therefore f$ changes concavity $2n$ times.

32. Vert asymp at $x = 2$ and $f' > 0$ means: 

Ⓑ

Thus $\lim_{x\to 2^-} f(x) = \infty$, $\lim_{x\to 2^+} = -\infty$, $\lim_{x\to 2} f(x) \to$ DNE

TI-83 33. Graph $f'(x)$ on $[0.2, 2.6]$ crosses the x-axis three times Ⓓ

34. To grow faster than e^x requires a larger base. Ⓓ

TI-83 35. Graph $f'(x)$ on $[0, 20]$ crosses the x-axis six times. Ⓒ

TI-83 36. $A = \frac{1}{2}\int_0^{\pi}[f(\theta)]^2 d\theta = 6.283 \approx 2\pi$ Ⓓ

37. $P_{3,2}(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$ Ⓔ

$\therefore f(2.2) \approx 7.20 + 3.5(.2) + \frac{1.75}{2}(.2)^2 + \frac{0.85}{6}(.2)^3$

$M = 1$ as the max value of $f^{(5)}$

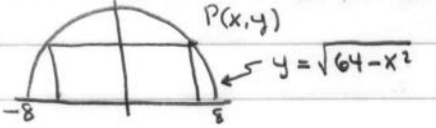
TI-83 | 38. $R_5(x) \le \left|\frac{M(.5)^5}{5!}\right| = .0002604 < .0003$ (B)

* TI-83 | 39. Graph y on $[0,1]$, count the zeros: There are many near $x=0$ because y oscillates. (E)

* TI-83 | 40. Want $g'(0)$ where $g = f^{-1}$. Solve $f(x) = 0 \Rightarrow x = 3$. Graphically, $f'(3) = 10$. $\therefore g'(0) = \frac{1}{10}$ (C)

41. $\frac{dy}{dx} = xy - y^2 \Rightarrow y(2) \approx y(1) + \left.\frac{dy}{dx}\right|_{\substack{x=1\\y=3}}(2-1) = 3 + (3-9)(1) = -3$ (A)

* TI-83 | 42.



$A = 2xy = 2x\sqrt{64-x^2} = 2x(64-x^2)^{1/2}$ (E)

$\frac{dA}{dx} = 2(64-x^2)^{1/2} + 2x \cdot \frac{1}{2}(64-x^2)^{-3/2}(2x)$

$= 2(64-x^2)^{-3/2}(64-2x^2)$

$= 0$ when $x^2 = 32 \Rightarrow x = 4\sqrt{2}$ $\therefore A = 2 \cdot 4\sqrt{2}(64-32)^{1/2} = 64$

(Can be done w/o calculus: graph $A = 2x\sqrt{64-x^2}$ on $[0,8]$ and find the maximum).

43. $\int x^n \sin x \, dx = -x^n \cos x + \int n x^{n-1} \cos x \, dx$ Tabular method! (A)

44. $\frac{dy}{dt} = \frac{2y}{t(t+2)} \Rightarrow \frac{1}{y}dy = \frac{2}{t(t+2)}dt \Rightarrow \ln|y| = \int \frac{2}{t(t+2)}dt$ (E)

$\Rightarrow \ln|y| = \int \frac{1}{t} - \frac{1}{t+2}\,dt$ (integration by partial fractions)

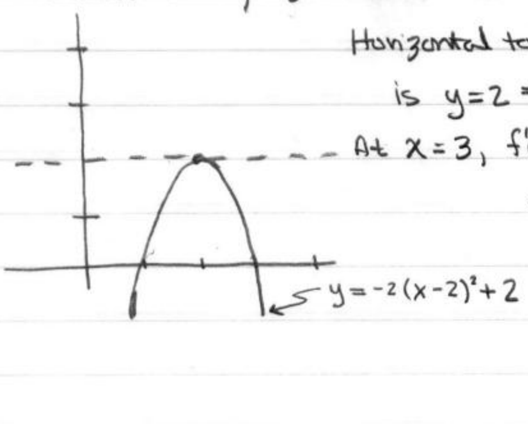
$\Rightarrow \ln|y| = \ln\left|\frac{t}{t+2}\right| + C$ when $t=1, y=1$

$\Rightarrow 0 = \ln\left|\frac{1}{3}\right| + C \Rightarrow C = -\ln\left(\frac{1}{3}\right) = \ln 3$

$\therefore \ln|y| = \ln\frac{t}{t+2} + \ln 3 = \ln\frac{3t}{t+2} \Rightarrow y = \frac{3t}{t+2}$

When $t=2$, $y = \frac{3 \cdot 2}{2 \cdot 2} = \frac{3}{2}$

* TI-83 | 45.



Horizontal tangent line through vertex is $y = 2 \Rightarrow b = 2$ (C)

At $x = 3$, $f'(3) = -4$ $\therefore y = -4(x-3) \Rightarrow b = 12$

$\therefore 2 \le b < 12$