AB Sample Exam V

1. $\frac{d}{dx}(2x^2+1)^4 = 4(2x^2+1)^3 \cdot 4x = 16x(2x^2+1)^3$ Ⓔ

2. $\int x\sqrt{x^2+1}\,dx$ $\quad \begin{matrix} u = x^2+1 \\ du = 2x\,dx \end{matrix} \rightarrow \frac{1}{2}\int u^{1/2}du = \frac{1}{3}u^{3/2} + C_1 = \frac{1}{3}(x^2+1)^{3/2} + C$ Ⓒ

3. $\frac{d^2y}{dx^2} = \frac{d}{dx}(2xy) = 2\cdot y + 2x\cdot y' = 2y + 2x\cdot 2xy = 2y + 4x^2y$ Ⓔ

4. $y = 2x^3 + 24x - 18 \rightarrow y' = 6x^2 + 24 = 6(x^2+4) > 0$ for all x. Ⓐ

5. $f(x) = \begin{cases} x^2+2 & x \le 1 \\ 2x+1 & x > 1 \end{cases}$ $\lim_{x\to 1^-} f(x) = 3$; $\lim_{x\to 1^+} f(x) = 3$, so $f(x)$ is Ⓒ

continuous. $f'(x) = \begin{cases} 2x & x \le 1 \\ 2 & x > 1 \end{cases}$ $\lim_{x\to 1^-} f'(x) = 2$ & $\lim_{x\to 1^+} f'(x) = 2$,

so $f(x)$ is differentiable at $x=1$ $\therefore f'(1) = 2$.

6. $f'(x) = 6x - 3x^2 \Rightarrow f''(x) = 6 - 6x = 0$ when $x = 1$. $\therefore f'(1) = 3$ Ⓓ

7. I. $\frac{d}{dx}\int_0^3 f(x)\,dx = 0$, so I is false ($2^{nd}$ FTC). Ⓑ

II. $\int_3^x f'(x)\,dx = f(x) - f(3)$, so II is false (FTC)

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$. (FTC), so III is true.

8. $\sin(xy) = x^2 \Rightarrow \frac{d}{dx}\left(\sin(xy) = x^2\right) \Rightarrow \cos(xy)(y + x\cdot y') = 2x$ Ⓔ

$\Rightarrow y\cdot\cos(xy) + x\cdot\cos(xy)\cdot y' = 2x \Rightarrow y' = \frac{2x - y\cos(xy)}{x\cos(xy)} = \frac{2x\sec(xy) - y}{x}$

9. Avg. vel. $= \frac{1}{b-a}\int_a^b v(t)\,dt = \frac{1}{b-a}\left[x(b) - x(a)\right]$. Here, $a = 0$, Ⓔ

so we want Avg. vel. $= 0 = \frac{1}{b}x(b)$ since $x(0) = 0, \Rightarrow x(b) = 0$:

$x = 3t^3 - 18t^2 + 24t = 3t(t^2 - 6t + 8t) = 3t(t-2)(t-4) = 0 \Rightarrow t = 0, 2, 4$

10. $y' = 12x^5 + 45x^4 + 40x^3 - 1 \Rightarrow y'' = 60x^4 + 180x^3 + 120x^2$ Ⓒ

$= 60x^2(x^2 + 3x + 2) = 60x^2(x+1)(x+2)$:

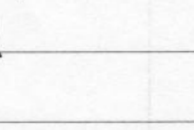
$60x^2$	+ 0 + + + + +
$x+1$	− − − 0 + + +
$x+2$	− − − − − 0 +
y''	+ + + 0 − 0 +

y'' changes sign two times $\Rightarrow$ 2 P.O.I.

11. $x^2 - 6x + 9 = y$:

x	0	1	2	3	4
y	9	4	1	0	1

"Inscribed"



$\int \approx 4 + 1 = 5$ Ⓓ

12. $y = \sin(2x) \rightarrow y' = 2\cos(2x) \rightarrow y'' = -4\sin(2x) \rightarrow y''' = -8\cos(2x)$ Ⓑ

$\rightarrow y^{IV} = 16\sin(2x)$ $\therefore$ Even derivs : $y^{2n} = (-2)^n\sin(2x)$ $\therefore y^{20} = 2^{20}\sin(2x)$

[Incidentally, odd derivs are $y^{(2n+1)} = (-2)^{2n+1}\cos(2x)$]

13. $f'(x) = 7 - 2x = 3 \Rightarrow x = 2$. $f(2) = 7(2) - (2)^2 = 10$ Ⓑ

$\therefore y - 10 = 3(x - 2) \Rightarrow y = 3x + 4$

14. The prompt only tells us that $f'(c)=0$ and c is between a and b. I. $f(a)$ does not necessarily equal $f(b)$ – this is not Rolle's thm. II. c could be a P.O.I and not an extremum (think $y=x^3$ @ $x=0$). III. c need not be a P.O.I. Ⓐ

15. The velocity graph looks something like:  Ⓓ

Acceleration is the derivative of v and looks like 

16. $f(x) = x\sqrt[3]{x} = x \cdot x^{1/3} = x^{4/3} \Rightarrow f'(x) = \frac{4}{3}x^{1/3}$ Ⓒ

17. $\int_k^6 \frac{dx}{x+2} = \Big[\ln|x+2|\Big]_k^6 = \ln 8 - \ln|k+2| = \ln k \Rightarrow \frac{8}{k+2} = k$ Ⓑ

$\therefore 8 = k^2+2k \Rightarrow 0 = k^2+2k-8 = (k+4)(k-2) \Rightarrow k = \cancel{-4}$ or 2

18. $\int_e^{e^2} \frac{dx}{x\ln x}$ $\quad u=\ln x$, $du = \frac{1}{x}dx$ $\quad x=e \to u=1$, $x=e^2 \to u=2$ $\Rightarrow \int_1^2 \frac{1}{u}du = \Big[\ln|u|\Big]_1^2 = \ln 2 - \ln 1 = \ln 2$ Ⓐ

19. $g(x) = f(3x) \Rightarrow g'(x) = f'(3x)\cdot 3 \Rightarrow g'(0.1) = 3f'(0.3) = 3(1.096) = 3.288$ Ⓔ

20. $\frac{d}{dx}\frac{8x+k}{x^2} = \frac{8(x^2)-(8x+k)\cdot 2x}{x^4} = \frac{-8x^2-2kx}{x^4} = \frac{-8x-2k}{x^3} = 0$ when $k=-4x$ Ⓑ

$\therefore k = -4(4) = -16$ when $x=4$.

21. The prompt is the definition of the derivative applied to Ⓓ

$f(x) = 2x^5 - 5x^3 \Rightarrow f'(x) = 10x^4 - 15x^2$

22. $\int_8^4 f(x)\,dx = -\int_4^8 f(x)dx = -\Big[\int_2^8 f(x)dx - \int_2^4 f(x)dx\Big] = -\big[-10-6\big] = 16$ Ⓔ

23. $y' = 3x^2+2ax+b \Rightarrow y'' = 6x+2a = 0$ at $x=2 \Rightarrow 12+2a=0 \Rightarrow a=-6$. Ⓓ

Then $y = x^3-6x^2+bx-8$ @ $(2,0) \Rightarrow 0 = (2)^3-6(2)^2+2b-8 \Rightarrow 24=2b \Rightarrow b=12$

24. $y = x^{-\frac{1}{3}} \to x = y^{-\frac{1}{3}} \to x^{-3} = y^{-1} \Rightarrow (y^{-1})' = -3x^{-4}$ Ⓔ

25. I & II are true (except perhaps in the debatable case of $f(x)=$ constant) Ⓒ

III is not necessarily true (think absolute value type functions).

26. Only graph C could fit in the contours suggested by the slope field. Ⓒ

27. $V(x) = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt}$. When $x=10$, $20 = 3(10)^2\cdot\frac{dx}{dt}$ Ⓔ

$\Rightarrow \frac{dx}{dt} = \frac{20}{300} = \frac{1}{15}$. $SA = 6x^2 \Rightarrow \frac{dSA}{dt} = 12x\frac{dx}{dt} = 12(10)\cdot\frac{1}{15} = 8\ \frac{cm^2}{sec}$

28. $\frac{dy}{dx} = \frac{x}{y} \Rightarrow \int y\,dy = \int x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2}+C$ or $y^2-x^2=C$. At Ⓐ

$(3,4)$, we have $4^2-3^2 = 7 = C$ $\therefore y^2-x^2=7$ or $x^2-y^2=-7$

Calculator Active

29. I. $A=\int_0^a f(x)\,dx$ is true. II. $A=\int_0^b f^{-1}(x)\,dx$ is true. (E)

III. $A=\int_0^b f^{-1}(y)\,dy$ is the same thing as II.

TI-83 30.  ← $x=1.170$ $\Rightarrow A=\int_0^{1.170}(3\cos x - x)\,dx = 2.0778$ (C)

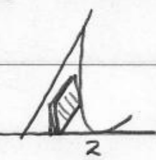
31. f' is a cubic : $f' = +1x^3 + \text{stuff} \Rightarrow f=\int f'\,dx = \frac{1}{4}x^4 + \text{stuff}$. (B)

So, f is a quartic with positive leading coefficient.

32. Horiz asy is y as $x\to\infty$: $\lim_{x\to\infty}\frac{2-e^{1/x}}{2+e^{1/x}} = \frac{2-e^0}{2+e^0}=\frac{2-1}{2+1}=\frac{1}{3}$ (C)

TI-83 33. Graph f' on $[2,10]$ (Zoom-0) : I. "Monotonic" means always non-decreasing or always non-increasing; f' changes sign, so I is false; (D)

II. true : f' changes from $-$ to $+$; III true : f' has 3 extrema.

TI-83 34.  $dV = (\text{area})(\text{tht}) = y^2\cdot dx = 9(x-2)^4\,dx$ (E)

$\therefore V = 9\int_0^2 (x-2)^4\,dx = 57.6$

TI-83 35. $y=\sin^2 x \to \frac{dy}{dx} = 2\sin x\cdot\cos x \big|_{x=\pi/4} = 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}=1=\frac{dx}{dx}\Rightarrow k=1$ (C)

36. compare $\frac{b-a}{2n}\left[f(a)+2f(a+\Delta x)+2f(a+2\Delta x)+2f(a+3\Delta x)+f(b)\right]$ with (D)

$\frac{1}{4}\left[\sqrt{1}+2\sqrt{\frac{5}{4}}+2\sqrt{2}+2\sqrt{\frac{13}{4}}+\sqrt{5}\right]\Rightarrow \frac{b-a}{n}=\frac{1}{2}=\Delta x$ We can see that $n=4 \Rightarrow b-a=2$. Therefore, choose D.

TI-83 37. $\frac{1}{\pi-1}\int_1^{\pi} e^{-x}\sin x\,dx = .1288$ (A)

TI-83 38. $y' = \frac{4}{3}\cos(4t)+\frac{1}{2}\sin(4t) \Rightarrow y'' = -\frac{16}{3}\sin(4t)+2\cos(4t)$ (A)

Graph y'' on $[0,5]$. The graph changes from $-$ to $+$ 3 times.

TI-83 39. New cost $= 14{,}500 + \int_0 (120+180\sqrt{t})\,dt = 16{,}441.64$ (C)

TI-83 40. $\frac{dy}{dt} = ky \Rightarrow \int \frac{1}{y}\,dy = \int k\,dt \Rightarrow \ln|y| = kt + C \Rightarrow y = Ae^{kt}$ (D)

$\frac{1}{2}$ present in 20 days $\Rightarrow \frac{A}{2} = Ae^{20k} \Rightarrow \ln\frac{1}{2} = 20k \Rightarrow k=\frac{1}{20}\ln\frac{1}{2} = -0.035$

TI-83 41. $f' = 2x\ln x + x^2\cdot\frac{1}{x} = 2x\ln x + x = 2 \Rightarrow x = 1.305$ solved graphically (A)

TI-83 42. The "fifth week of growth" is from $t=4$ to $t=5$: $\frac{m(5)-m(4)}{5-4} = 6.546$ g/wk (D)

43. I is true ($f'(2)>0$); II is true (f' changes from $-$ to $+$); III is true (f: 0 to 1 to 0) (E)

TI-83 44. $x'(t) = 3t^2+22t+6\pi\sin(\pi t) > 0$ for all $t>0$ (graph) (E)

TI-83 45. $\lim_{n\to\infty}\sum_{i=1}^{n} x_i\cos(x_i)\Delta x = \int_0^{\pi} x\cos x\,dx = -2$ (A)