AB Sample Exam IV

1. $\lim_{x\to 6} \frac{6-x}{\sqrt{x}-\sqrt{6}}\cdot\frac{\sqrt{x}+\sqrt{6}}{\sqrt{x}+\sqrt{6}} = -\lim_{x\to 6}\frac{(x-6)(\sqrt{x}+\sqrt{6})}{x-6} = -2\sqrt{6}$ (A)

2. $y' = \sin x + 2\sec^2(2x)\Big|_{x=0} = 2 \quad \therefore y-1=2(x-0)$ or $y=2x+1$ (B)

3. A is false : $\lim_{x\to 3} f(x)=3$; B is false : $\lim_{x\to 4} f(x)$ DNE ; C is false : $\lim_{x\to 3} f(x)$ $\neq f(3)$; E is false : $f(x)$ is not continuous at $x=6$; D is true (D)

4. $f'(x) = 0 + \frac{x-2}{|x-2|}\Big|_{x=2} = \frac{2-2}{|2-2|}$ which does not exist: a cusp! (E)

5. $\int(3x+5)^2dx \Rightarrow u=3x+5,\ du=3dx \Rightarrow \frac{1}{3}\int u^2du = \frac{1}{9}u^3+C_1=\frac{1}{9}(3x+5)^3+C$ (D)

6. $x(t)=\ln t$ avg vel is the average rate of change on $[1,e] = \frac{\ln e-\ln(1)}{e-1}$ (C)
$= \frac{1}{e-1}$

7. $e^{xy}=2 \Rightarrow \ln e^{xy}=\ln 2 \Rightarrow xy=\ln 2 \Rightarrow \frac{d}{dx}(xy)=\frac{d}{dx}\ln 2$ (A)
$\Rightarrow y+x\cdot y'=0 \Rightarrow y'=\frac{-y}{x}\Big|_{\substack{x=1\\ y=\ln 2}} = -\ln 2$

8. $f(x)=3x\ln x \Rightarrow f'(x)=3\ln x+3x\cdot\frac{1}{x} = 3\ln x+3$ or $\ln x^3+3$ (A)

9. $\int_{-3}^{3}|x+2|dx = \int_{-3}^{-2}(-x-2)dx + \int_{-2}^{3}(x+2)dx = -\left(\frac{x^2}{2}+2x\right)\Big]_{-3}^{-2} + \left(\frac{x^2}{2}+2x\right)\Big]_{-2}^{3}$ (C)
$= -(2-4)+\left(\frac{9}{2}-6\right)+\left(\frac{9}{2}+6\right)-\left(\frac{4}{2}-4\right) = 13$

10. $y=x^2-2kx$ has x-int at $x=0$ and $x=2k$. Area $=\left|\int_0^{2k} x^2-2kx\,dx\right|$ (B)
$=\left|\frac{x^3}{3}-kx^2\right|\Big]_0^{2k} = \left|\frac{8k^3}{3}-4k^3\right| = \frac{4k^3}{3}=36 \Rightarrow k^3=27 \therefore k=3$

11. $f'(x)=e^x-2=0$ when $x=\ln 2$. Then $f(\ln 2)=e^{\ln 2}-2\ln 2=2-2\ln 2$ (D)

12. MVT: $f'(c)=\frac{f(7)-f(-3)}{7-(-3)}=\frac{2-4}{10}=-\frac{1}{5}$ (D)

13. $y=\frac{1-x}{x-1}=-1 \quad \therefore \frac{dy}{dx}=0$ except at $x=1$ (B)

14. $g(x)=\frac{1}{f(x)}=[f(x)]^{-1} \Rightarrow g'(x)=\frac{-1}{[f(x)]^2}\cdot f'(x) \Rightarrow g'(2)=\frac{-1}{[f(2)]^2}\cdot f'(2)=\frac{-1}{64}\cdot -4=\frac{1}{16}$ (C)

15. 

$\int_1^7 \ln x\,dx \approx [\ln 3+\ln 5+\ln 7](2)$ (C)

16. $S=4\pi r^2 \Rightarrow \frac{ds}{dt}=8\pi r\frac{dr}{dt}$ $V=36\pi=\frac{4}{3}\pi r^3 \Rightarrow r=3$. Also, (A)
$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt} \Rightarrow 12=4\pi(3)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=\frac{1}{3\pi}$. Then $\frac{ds}{dt}=8\pi\cdot 3\cdot\frac{1}{3\pi}=8$ ft/sec

17. $f(x)=15-g(x) \Rightarrow f(x)-g(x)=15-2g(x) \Rightarrow \int_{-2}^{2}[15-2g(x)]dx=60-2\int_{-2}^{2}g(x)dx$ (E)

18. $\int_2^4\left[\frac{d}{dt}(3t^2+2t-1)\right]dt = (3t^2+2t-1)\Big]_2^4 = [3(16)+2(4)-1]-[3(4)+2(2)-1]=40$ (B)

19. I is true since there must be at least one root on $(0,a)$. (D)

II is true because P is odd degree.

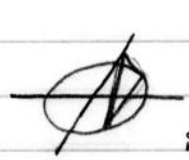
III is false because $\int_0^a P(t)dt$ is the displacement.

20. I is true since we have a min. on a differentiable function. (A)

II may not be true: we don't know if f is twice differentiable

III is not true if II is false.

21. Base is $x^2+y^2=9 \Rightarrow y=\sqrt{9-x^2}$ (E)



$dV = (\text{area})(\text{thk}) = \frac{\sqrt{3}}{4}S^2\cdot(\text{thk})$ $\quad S = 2\sqrt{9-x^2}$

$dV = \sqrt{3}(9-x^2)dx \Rightarrow V = 2\sqrt{3}\int_0^3 (9-x^2)dx = 2\sqrt{3}\left(9x-\frac{x^3}{3}\right)\Big|_0^3 = 36\sqrt{3}$

22. $u = 25-x^2 \Rightarrow du = -2x\,dx \quad x=0 \to u=25,\ x=3\to u=16$ (D)

$\therefore \int_0^3 x\sqrt{25-x^2}\,dx = \frac{1}{2}\int_{16}^{25}\sqrt{u}\,du$

23. $y=\sin^{-1}u \Rightarrow y' = \frac{1}{\sqrt{1-u^2}}\cdot u' \quad \therefore \frac{1}{\sqrt{1-(3x/4)^2}}\cdot\frac{3}{4} = \frac{3}{\sqrt{16-9x^2}}$ (E)

24. Increasing at a decreasing rate means concave down. Choose... (C)

25. $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \int\frac{1}{y}dy = \int\frac{2}{x}dx \Rightarrow \ln|y| = 2\ln|x| + C = \ln x^2 + C \Rightarrow y = Ae^{\ln x^2} = Ax^2$ (B)

26. $\lim_{h\to 0}\frac{\tan(2(x+h)) - \tan(2x)}{h} = \frac{d}{dx}\tan(2x) = \sec^2(2x)\cdot 2$ (D)

27. $A = \int_0^{2\pi}\sin\left(\frac{x}{2}\right)dx = -2\left[\cos\frac{x}{2}\right]_0^{2\pi} = -2\left[\cos\pi - \cos 0\right] = 4$ (C)

28. $f'' = \begin{cases} 2x & x\le 2 \\ \frac{1}{2} & x>2\end{cases} = 0$ only at $x=0$ (A)

Calculator active: TI-83

29. Sphere is generated by revolving the semi-circle $y=\sqrt{4-x^2}$ (C)

$\therefore dV = \pi R^2(\text{thk}) = \pi\left(\sqrt{4-x^2}\right)^2dx = \pi(4-x^2)dx \quad \therefore V = 2\pi\int_0^2(4-x^2)dx$

TI-83 30. Speed $= |v(t)| = \left|\frac{dx}{dt}\right| = |4t^3 - 30t^2 + 58t - 36|$, which has its max at $t=4$ (D)

31. f is always increasing, concave up for $x<0$, concave down for $x>0$. Choose... (E)

TI-83 32. The graphs intersect at $x=0$ where both have horizontal tangent lines. (A)

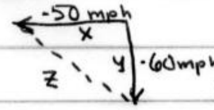
TI-83 33. $\int_1^{10}(2-\cos x)dx \approx \frac{1}{2}[2-\cos(1)+2-\cos(5)](5-1) + \frac{1}{2}[2-\cos(5)+2-\cos(8)](8-5)$ (B)

$+\frac{1}{2}[2-\cos(8)+2-\cos(10)](10-8) = 17.129$

TI-83 34. $v(t) = 2$ when $t=0$; $v(t)=4$ when $e^{2t}=2 \Rightarrow t=\ln\sqrt{2}$. The (A)

distance traveled is $2\int_0^{\ln\sqrt{2}}e^{2t}dt = 1$

TI-83 35. Graph f' on $[0,12]$. f' changes from + to − two times. (C)

36. 

$z^2 = x^2+y^2 \Rightarrow \cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$ (D)

@ $t = 30\,min$, $x=-25$, $y=-30$, $z = \sqrt{(-25)^2+(-30)^2} = 39.051$

Then $39.051\cdot\frac{dz}{dt} = -25(-50) - 30(-60) \Rightarrow \frac{dz}{dt} = 78.102\,mph$

TI-83 37. Area under the parabola is $\int_0^2(-x^2+2x)dx = \frac{4}{3}$. Area of square is 4. (E)

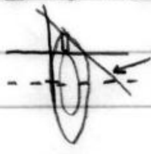
Area above parabola is $4-\frac{4}{3} = \frac{8}{3}$. $\therefore$ Prob $= \frac{8/3}{4} = \frac{2}{3}$

TI-83 38. $\frac{df}{dx} = \frac{d}{dx}\int_0^x \text{Arcsin}\,t\,dt = \arcsin x \Big|_{x=0.4} = .412$ Ⓒ

TI-83 39. $\frac{dg}{dx} = \frac{d}{dx}\int_3^x ((5+4t-t^2)(2^{-t}))dt = (5+4x-x^2)(2^{-x})$; graph shows Ⓐ

that $g' = 0$ at $x = 5$ and -1 only : g' $\underset{-1}{-\;0}\;+\;\underset{5}{0}\;-$

I is true ; II is false ; III is false by comparing areas.

TI-83 40.  $x+y=1$ or $y=1-x$ $dV = \pi(R^2 - r^2)(thk) = \pi((1-x-(-1))^2 - (+1)^2)dx$ Ⓔ

$\Rightarrow V = \pi\int_0^1((2-x)^2-1)dx = \frac{4\pi}{3}$

41. $\frac{dy}{dx} = \cos x - \sin x$ I: $y + \frac{dy}{dx} = \sin x + \cos x + \cos x - \sin x = 2\cos x$ False Ⓔ

II, $y + \frac{dy}{dx} = 2\cos x$ True. III $\frac{dy}{dx} - y = \cos x - \sin x - (\sin x + \cos x) = -2\sin x$ True

TI-83 42. Shell method! $dV = 2\pi(\text{rad})(\text{ht})(\text{thk}) = 2\pi \cdot x \cdot k(x-2)^2\,dx$ Ⓔ

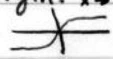
$\therefore V = 2\pi\int_0^2 kx(x-2)^2 dx = 2k\pi\left[\frac{4}{3}\right] = 8\pi \Rightarrow k = 3$

TI-83 43. $y^2 - x^3 - 15x^2 = 0 \Rightarrow y = \pm\sqrt{x^3+15x^2} \Rightarrow y' = \pm\frac{1}{2}(x^3+15x^2)^{-\frac{1}{2}}(3x^2+30x)$ Ⓐ

$= \pm\frac{3}{2}(x^3+15x^2)^{-1/2}\cdot x(x+10) = 0$ at $x = 0, -10$. Reject $x=0$

since it is a double cusp : 

44. Odd $\Rightarrow$ symmetry over the origin: $\lim_{x\to-\infty} f(x) = -3$ Ⓔ

I : $\lim_{x\to+\infty} f(x) = 3$ is true 

II: True; vertical asymptotes violate continuity.

III: True if the initial condition is true.

45. Average value of 0 on symmetric limits : $\frac{1}{a-(-a)}\int_{-a}^{a} f(x)dx = 0$ Ⓐ

implies odd function symmetry : choose...