AB Sample Exam III

1. $A=\int_1^3 (3x^2+2x)\,dx = \left[x^3+x^2\right]_1^3 = (3^3+3^2)-(1^3+1^2) = 34$ (B)

2. $\lim_{x\to3}\frac{(3-x)^2}{(x-3)} = \lim_{x\to3}(3-x)=0$ (C)

3. $C(s)$ is in gal/mi and s ($\Rightarrow ds$) is in mi/hr $\therefore \int_a^b c(s)\,ds$ is in $\frac{gal}{mi}\cdot\frac{mi}{hr}=\frac{gal}{hr}$ (C)

4. $V_{sphere}=\frac43\pi r^3 \Rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$. Then $\frac{dV}{dt}=4\pi(10)^2\cdot 2 = 800\pi$ in^2/sec (D)

5. $v(t)=t^2 \Rightarrow x(3)-x(1)=\int_1^3 t^2\,dt=\left[\frac{t^3}{3}\right]_1^3=\frac{27}{3}-\frac13=\frac{26}{3}$ (B)

6. $\frac{d}{dx}\left[(4x)^3\cdot(2x)^6\right] = 3(4x)^2\cdot4\cdot(2x)^6+(4x)^3\cdot6(2x)^5\cdot2)$ (D)

$=(4x)^2(2x)^5\left[3\cdot4\cdot2x+4x\cdot6\cdot2\right]=(4x)^2(2x)^5(24x+48x)=72x(4x)^2(2x)^5$

7. I is true; the contours for $y>2$ flow to the horiz. asymp $y=2$. (D)

II is true; " " " $y>0$ " " " " " "

III is false; " " " $y<0$ flow to $y\to-\infty$.

8. $\int_1^3 \frac{x}{x^2+1}\,dx=\frac12\int_2^{10}\frac1u\,du$ $u=1+x^2$ $du=2x\,dx$ $x=1\to u=2$; $x=3\to u=10$ (D)

$=\frac12\ln|u|\Big]_2^{10}=\ln\sqrt{10}-\ln\sqrt2=\ln\frac{\sqrt{10}}{\sqrt2}=\ln\sqrt5$ or $\frac12\ln5$

9. $\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right)=\frac{d}{dx}\ln(x^2-1)^{-1}=-\frac{d}{dx}\ln(x^2-1)=\frac{-1}{x^2-1}\cdot2x=\frac{-2x}{x^2-1}$ (B)

10. $\int 2\tan x\,dx = 2\ln|\sec x| = \ln(\sec^2 x)$ (C)

11. $u=x^2+1\Rightarrow x^2=u-1$ $du=2x\,dx \Rightarrow x\,dx=\frac12du$ (A)

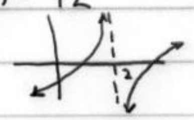
Then $\int_0^2\frac{x^3}{x^2+1}\,dx=\frac12\int_1^5\frac{u-1}{u}\,du=\int_1^5\frac{u-1}{2u}\,du$

12. $y'=\frac{k(k+x)-1(kx+8)}{(k+x)^2}=\frac{k^2+\cancel{kx}-\cancel{kx}-8}{(k+x)^2}\Big|_{x=-2}=\frac{k^2-8}{(k-2)^2}=1$ (slope of $y=x+4$) (D)

$\Rightarrow k^2-4k+4=k^2-8 \Rightarrow -4k=-12\Rightarrow k=3$

13. $\int_5^{12}f(x)\,dx\approx 7(6-5)+11(9-6)+12(11-9)+8(12-11)=72$ (E)

14. $f'>0$ and vert asym @ $x=2 \Rightarrow$ Pos. graph (C)



I. is false: $\lim_{x\to2^-}f(x)\neq\lim_{x\to2^+}f(x)$

II. is false: $\lim_{x\to2^+}f(x)=-\infty$ III. is true

15. $y=5^{x^3-2}\Rightarrow \frac{dy}{dx}=\ln5\cdot5^{x^3-2}\cdot3x^2$ (B)

16. Here $\Delta x\leftrightarrow\frac1n$, so we are adding $\sin\pi\cdot\Delta x+\sin\pi\cdot2\Delta x+\cdots+\sin\pi$

which is the Riemann Sum for $\int_0^1 \sin\pi x\,dx = \frac1\pi\left[-\cos\pi x\right]_0^1=\frac2\pi$ (B)

17. f' shows that f decreases for $x<2$ and increases from $x=2$ to ∞ with (D)

a P.O.I. at $x=1$. Choose...

18. $\frac{d}{dx}\int_x^0\frac{du}{1+u^2}=0-\frac{1}{1+x^2}$ (2nd FTC) (B)

19. MVT: $\frac{f(10)-f(-2)}{10-(-2)}=\frac{2-26}{10+2}=-2$. $f'=-2$ at $(4,23)$ $\therefore y-23=-2(x-4)$ (C)

$\Rightarrow y=-2x+31$

20. $\frac{dy}{dx} = 4xy \Rightarrow \frac{1}{y}dy = 4x\,dx \Rightarrow \ln|y| = 2x^2 + C \Rightarrow |y| = Ae^{2x^2}$ Ⓒ

Using $(0,4)$, $|4| = Ae^0 \Rightarrow A = 4 \quad \therefore y = 4e^{2x^2}$

21. $y' = 3x^2 - 12x \Rightarrow y'' = 6x - 12 = 0$ @ $x = 2$. Then $y'(2) = -12$ Ⓐ

and $y(2) = -16 \quad \therefore$ Tangent line: $y + 16 = -12(x-2)$ or $y = -12x + 8$

22. $\int_0^k \cos(2x)dx = \frac{1}{2}\sin(2x)\Big]_0^k = \frac{1}{2}\sin(2k) = \frac{1}{2} \Rightarrow \sin(2k) = 1$ Ⓑ

$\Rightarrow 2k = \frac{\pi}{2} \Rightarrow k = \frac{\pi}{4}$

23. I & III are true ; II is not true because x is not constant. Ⓒ

24. $f(x) = (e^{2x}+1)^{1/2} \Rightarrow f'(x) = \frac{1}{2}(e^{2x}+1)^{-1/2} \cdot e^{2x} \cdot 2$

$\therefore f'(0) = \frac{1}{2}(e^0+1)^{-1/2} \cdot e^0 \cdot 2 = 2^{-1/2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ Ⓒ

25. $x^2y + yx^2 = 6 \Rightarrow 2xy + x^2y' + x^2y' + 2xy = 0$ Ⓔ

$\Rightarrow 2x^2y' = -4xy \Rightarrow y' = \frac{-2y}{x}$ Then $y'' = \frac{-2xy' + 2y}{x^2} = \frac{4y + 2y}{x^2} = \frac{6y}{x^2}$

Then at $(1,3)$, $y'' = \frac{6(3)}{(1)^2} = 18$

26. 

$x + 2y = 4 \quad dV = (\text{area})(\text{thk}) = \frac{1}{2}\pi r^2(\text{thk}) = \frac{\pi}{2} \cdot \frac{1}{4}(2 - \frac{x}{2})^2 dx$ Ⓐ

$\Rightarrow y = 2 - \frac{x}{2} \quad \therefore V = \frac{\pi}{8}\int_0^4 (4 - 2x + \frac{x^2}{4})dx = \frac{\pi}{8}(4x - x^2 + \frac{x^3}{12})\Big|_0^4$

$= \frac{\pi}{8}(16 - 16 + \frac{16}{3}) = \frac{16\pi}{24} = \frac{2\pi}{3}$

27. $v(t) = \frac{dx}{dt} = (t-3)^3 + (t+1) \cdot 3(t-3)^2 = (t-3)^2(t - 3 + 3(t+1))$ Ⓓ

$= (t-3)^2(4t)$. $a(t) = \frac{dv}{dt} = 2(t-3)(4t) + (t-3)^2 \cdot 4$

$= 4(t-3)(2t + t - 3) = 4(t-3)(3t-3)$

$4(t-3)$	$-$	$-$	$-$	0	$+$
$3(t-1)$	$-$	0	$+$	$+$	$+$
		1		3	
a:	$+$	0	$-$	0	$+$

(v increasing $\Rightarrow a > 0$)

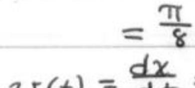
28. $\frac{dy}{dt} = ky \Rightarrow \frac{1}{y}dy = k\,dt \Rightarrow \ln|y| = kt + C \Rightarrow |y| = Ae^{kt}$ Ⓓ

Init. Pop $= A = 1500 \quad \therefore y = 1500e^{kt}$ Since the population quadruples in two days, it must double in one day. Therefore $e^k = 2$

And $y = 1500(2)^t$. A $t = 3$, $y = 1500(2)^3 \Rightarrow$ pop increases by $2^3 = 8$ times.

Calculator Active : TI-83

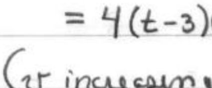
29. $\frac{1}{7-3}\int_3^7 f(x)dx = 12 \Rightarrow \int_3^7 f(x)dx = (7-3)(12) = 48$ Ⓔ

TI-83 30. 

$dV = (\text{area})(\text{thk}) = \pi R^2 \cdot \text{thk} = \pi(\cos(\cos x))dx$ Ⓒ

$\therefore V = \pi\int_0^{\pi} \cos(\cos x)dx = 6.039$

31. $h'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{[g(3)]^2} = \frac{(-1/3)(3) - (1)(1)}{(3)^2} = -\frac{2}{9}$ Ⓐ

TI-83 32. 

Shaded region's area $= \int_0^c \cos x\,dx = \frac{1}{2}\int_0^{\pi/2} \cos x\,dx = 1/2$ Ⓑ

$\Rightarrow \int_0^c \cos x\,dx = [\sin x]_0^c = \sin c - 0 = 1/2 \Rightarrow c = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

TI-83 (if solved graphically) 33. $f'(x) = 15 + 12x - 3x^2 = 3(5-x)(1+x) = 0$

		-1		5	
$3(5-x)$	$+$	$+$	$+$	0	$-$
$(1+x)$	$-$	0	$+$	$+$	$+$
f'	$-$	0	$+$	0	$-$

f decreases on $(-\infty,-1)$ and $(5,\infty)$ because $f'<0$ Ⓓ

TI-83 34. FTC: $f(1) - f(0) = \int_0^1 \frac{\tan^2 x}{1+x^2}dx \Rightarrow f(0) = \frac{1}{2} - \int_0^1 \frac{\tan^2 x}{1+x^2}dx = .155$ Ⓑ

TI-83 35. Graph f'' on $[0,20]$, count the zeros: there are six. Ⓒ

36. Linear approximations are over-estimates when f is concave down. Ⓒ

TI-83 37. Use 2^{nd}–Trace 6, test the choices after graphing $f(x)$ Ⓓ

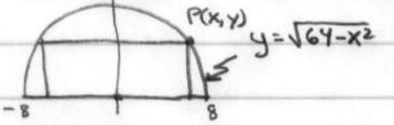
38. $\lim_{x\to 3} \frac{g(3)-g(x)}{3-x} = g'(3)$ (defn) Since $g'(3)<0$, $g(x)$ is decreasing Ⓐ

TI-83 39. Graph y on $[0,1]$, count the zeros: There are many near $x=0$ because y oscillates. Ⓔ

TI-83 40. $f(x) = 0$ when $x=3$. $\Rightarrow f'(3) = 10$ $\therefore (f^{-1})'(0) = \frac{1}{10}$ Ⓒ

41. $f(\ln x) = x^2 \Rightarrow f(x) = f(\ln e^x) = (e^x)^2 = e^{2x}$ Ⓔ

TI-83 42.



$A = 2xy = 2x\sqrt{64-x^2} = 2x(64-x^2)^{1/2}$ Ⓔ

$\frac{dA}{dx} = 2(64-x^2)^{1/2} + 2x\cdot\frac{1}{2}(64-x^2)^{-3/2}\cdot(-2x)$

$= 2(64-x^2)^{-3/2}(64-x^2-x^2) = 2(64-x^2)^{-3/2}(64-2x^2)$

$= 0$ when $x^2 = 32 \Rightarrow x = 4\sqrt{2}$ $\therefore A = 2\cdot 4\sqrt{2}(64-(4\sqrt{2})^2)^{1/2} = 64$

(can be done w/o calculus: graph $A = 2x\sqrt{64-x^2}$ on $[0,8]$ and find the maximum).

43. Differentiable $\Rightarrow f(x)$ & $f'(x)$ are continuous. Ⓒ

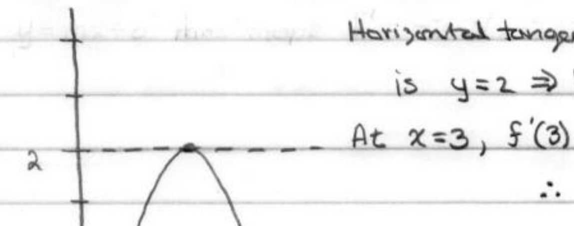
$f'(x) = \begin{cases} -e^{-x} & x<0 \\ a & x\geq 0 \end{cases} \Rightarrow -e^{-0} = a \Rightarrow a = -1$

$f(x) = \begin{cases} e^{-x}+2 & x<0 \\ -x+b & x\geq 0 \end{cases} \Rightarrow 1+2 = 0+b \Rightarrow b=3$

$\Bigg\}\ a+b=2$

TI-83 44. Since $g(x)$ dominates, graph $g(x)-f(x)$ on $[-10,10]$; we see two zeros. Then extend the window to $[-10,100]$; we see a third. Ⓓ

TI-83 45.



Horizontal tangent line through vertex is $y=2 \Rightarrow b=2$ Ⓒ

At $x=3$, $f'(3) = -4$ $\therefore y = -4(x-3) \Rightarrow b=12$

$\therefore 2 \leq b < 12$